1-EDGE MAGIC LABELING FOR SOME CLASS OF GRAPHS

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ABSTRACT
The purpose of this paper is to introduce new labeling called 1-Edge Magic Labeling and is to obtain the existence of this labeling for certain graphs.

KEYWORDS: *Graph Labeling, Edge Labeling, 0-Edge Magic Labeling.*

1. INTRODUCTION

By G (p, q) we denote a graph having p vertices and q edges, by V(G) and E(G) the vertex set and edge-set respectively. By a labeling we mean a one-to-one mapping that carries a set of graph elements into a set of numbers (usually integers), called labels. We deal with labeling with domain either the set of all vertices, or the set of all edges, or the set of all vertices and edges, respectively. We call these labeling a vertex labeling, or an edge labeling, or a total labeling, depending on the graph elements that are being labeled. The origin of this labeling is introduced by Rosa.
The concept of labeling of graphs has gained a lot of popularity in the area of graph theory. This popularity is not only due to mathematical challenges of graph labeling but also to the wide range of applications that graph labeling offer to other branches of science, for instance, X-ray, crystallography, coding theory, cryptography, astronomy, circuit design and communication networks design. In the last three decades magic and antimagic labeling, prime labeling, graceful labeling, k-graceful labeling, and odd labeling, even and odd mean labeling and strongly labeling etc. have been studied in over 1300 papers. Recently in 2012, J. Jayapriya and K.Thirusangu introduced 0-Edge Magic Labeling and shown the existence of this labeling for some class of graphs. Motivated by 0-Edge Magic Labeling we introduce 1-Edge Magic Labeling for certain graphs as for P_n, C_n, $G\odot K_1$, $S_{m\,n}$ (i.e. double star) etc.

2. PRELIMINARY

Before looking at vertex labeling and edge labeling, we first look at some basic concepts of graph theory.

Definition 2.1 The vertex-weight of a vertex v in G under an edge labeling is the sum of edge labels corresponding to all edges incident with v. Under a total labeling, vertex-weight of v is defined as the sum of the label of v and the edge labels corresponding to the entire edges incident with v. If all the vertices in G have the same weight k, we call the labeling vertex-magic labeling or vertex-magic total labeling respectively and we call k a magic constant. If all the vertices in G have different weights, then the labeling is called vertex-antimagic edge labeling or vertex antimagic total labeling.

Definition 2.2 The edge- weight of an edge e under a vertex labeling is defined as the sum of the vertex labels corresponding to every vertex incident with e, under a total labeling, we also add the label of e. Using edge –weight, we drive edge magic vertex or edge magic total labeling and edge-antimagic vertex or edge antimagic total labeling.

Definition 2.3 A (p, q) graph G is said to be (1 ,0) edge-magic with the common edge count k if there exist a bijection f:V(G)→{1,2,…………p} such that for all e=uv εE(G), f(u)+f(v)=k. It is said to be (1,0) edge-antimagic if for all e = uv εE(G), f(u)+f(v) are distinct.

Definition 2.4 A (p, q) graph G is said to be (0, 1) vertex –magic with the common vertex count k if there exist a bijection f : E(G)→ {1, 2,…………q } such that for each u ε V(G), Σ f(e)=k, where e ε E(G) and e is incident on u. It is said to be (0, 1) vertex-antimagic if all the weights are distinct.

Definition 2.5 A (p, q) graph G is said to be (1, 1) edge-magic with the common edge count k if there exist a bijection f : V(G)UE(G) → {1,2,…………….. p + q} such that f(u) + f(v) + f(e) =k for all e=uv εE (G). It is said to be (1, 1) edge antimagic if f(u) +f(v) +f(e) are distinct for all e = uv εE(G).

Definition 2.6 Let G =(V, E) be a graph , V={v_i ,1≤ i≤ n } and E = {$v_i v_{i+1}$, 1≤i≤i+1}. Let f : V →{-1, 1} and f* : E→ {0}, such that for all uv εE, f* (u v) =f(u)+f(v)=0 then the labeling is said to be 0-Edge Magic labeling.

Definition 2.7 G^+ =GΘK_1 is a graph obtained by joining exactly one pendant edge to every vertex of a graph G.

3. MAIN RESULTS

The concept of 0-Edge Magic Labeling motivate us to define the following new definition of 1-Edge Magic Labeling.

1-Edge Magic Labeling: Let G=(V, E) be a graph where V= { v_i, 1≤i≤n } and
E = {v_i v_{i+1},1≤i≤n}

Let f : V → {-1, 2} and f* : E → {1} such that for all uv εE, f*(u v)= f(u) + f(v) =1 then the labeling is said to be 1-Edge Magic Labeling.

Using this new definition, we prove some results as follows:

Theorem 3.1 P_n admits 1-Edge Magic Labeling for all n.

Proof. Let G =(V, E) be a graph where V= {v_i , 1≤i≤n } and E = {v_i v_{i+1} ,1≤i≤n}.

Let f : V → {-1, 2 }

Such that f(v_i) =-1 if i is odd.[1]

f(v_{i+1}) =2 if i is even

1≤i≤n

f* (v_i. v_{i+1}) = -1+2=1 if i is odd.

f* (v_i. v_{i+1}) =2-1=1 if i is even

Hence p $_n$ admits 1-Edge Magic Labeling.

Theorem 3.2 C_n admits 1-Edge Magic Labeling when n is even.

Proof. Let G =(V, E) be a graph where V={v_i : 1 ≤ i ≤ n} and E={ v_i. v_{i+1} , 1 ≤ i ≤ n}.

Let f ; V→{-1,2 }such that

f(v_i)= -1 if i is odd

f(v_i)= 2 if i is even

1≤i≤n

f * (v_i. v_{i+1}) = -1+2=1 if i is odd.

f* (v_i. v_{i+1}) =2-1=1 if i is even,

Hence C_n admits 1-Edge Magic Labeling for every even n.

Example 1: To support the Theorem 3.2 we give an example of 1-Edge Magic Labeling for C_6.

Solution:

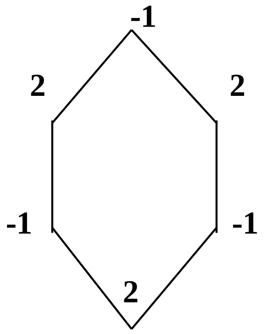

Fig 1:- 1- Edge Magic labeling for c_6.

Theorem 3.3 If G admits 1- Edge Magic Labeling then $G\Theta K_1$ admits 1- Edge Magic Labeling.

Proof :- Let $G=(V, E)$ be a graph where $V=\{ v_i : 1 \le i \le n \}$ and $E=\{ v_i . v_{i+1} , 1 \le i \le n\}$.

Let $f : V \rightarrow \{-1,2 \}$ and $f^* : E \rightarrow \{1\}$ such that for all $v_i . v_{i+1} \in E$

$f^* (v_i . v_{i+1}) = f (v_i) + f (v_{i+1}) = 1$

Let

$G\Theta K_1 = (V ,E) \cup \{ v_j : 1 \le j \le n \} \cup \{ v_i . v_j : 1 \le i , j \le n \}$, then let $g: V \rightarrow \{-1, 2\}$ and $g^* : E \rightarrow \{1\}$, then for all $v_i . v_{i+1} \in E$.

Let $v_j = -1$ if $v_{Ii} = 2$ or $v_j = 2$ if $v_i = -1$ then $g^*(v_i v_j) = g^*(v_i) + g^*(v_j) = 1$

Hence the theorem.

Corollary: Ladder, Wheel graph, K_n admits 1- edge magic labeling if $n \equiv 0 \pmod 2$ i.e. n is even.

Example 2: To support the Theorem 3.3 we give example of 1-Edge Magic Labeling for $P_4\Theta K_1$

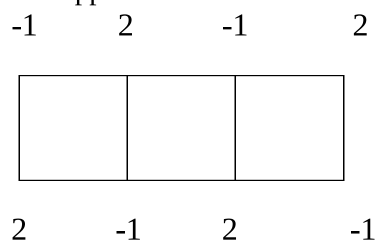

Figure 2:- 1-Edge Magic Labeling for $P_4\Theta K_1$.

Theorem 3.4 : Let $G = S_{m,n}$ be a double star graph then G admits 1- Edge Magic Labeling.

Proof : Let G = (V, E) be a double star graph denoted by $S_{m,n}$ and v_1 and v_2 are two vertices in $S_{m,n}$ which are not pendent. Let u_i' s are m pendent vertices to v_i and u_j' are n pendent vertices to v_2.

Let $f: V \rightarrow \{-1 , 2\}$

such that $f (v_i) = -1$ and $f (v_2) = 2$

and $f(u_i) = 2$ for $1 \le i \le m$

and $f(u_j) = -1$ for $1 \le j \le n$

$f^*(v_1 u_i) = -1 + 2 = 1$ if $1 \le i \le m$

Also $f^*(v_2 u_j') = 2 - 1 = 1$ if $1 \le j \le n$

Hence the proof.

Example 3: To support the Theorem 3.4 we give example of 1-Edge Magic Labeling for $S_{4,5}$.

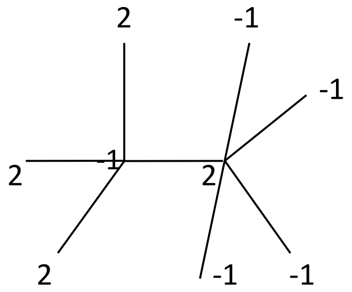

Figure 3:- 1-Edge Magic Labeling for $S_{4,5.}$

4. CONCLUSION

In this paper we have investigated some class of graphs which admits 1- Edge Magic Labeling. Further investigation can be done to obtain the conditions at which some graphs admits 1-Edge Magic Labeling.

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